CHAPTER : RATIO AND PROPORTION

TOPIC : INTRODUCTION OF RATIO AND PROPORTION

Date : 4 Dec 2020

Points to discuss -

(i) Introduction

(ii) Ratio of Two Quantities in the Same Unit

(iii) To Convert a Given Ratio to its Simplest Form

(iv) Equivalent Ratio

(v) To divide a number into two or more parts in a given ratio

(vi) Comparison of Ratios

(v) Proportion

(vi) Unitary Method / Word Problems

(a) Direct Change

(b) Indirect Change

Ratio of Two Quantities in the Same Unit - Two quantities can be compared only when they are of the same kind and measured in the same unit. It is meaningless to compare animals and human beings or weights in grams and kilograms.

We can compare weights only if they are measured in the same units. (They can be converted into the same units for comparison.) When two similar quantities are expressed as a ratio, it is a pure number having no unit.

Therefore, the **ratio has no unit.**

Ratio of Quantities with the Same Units –

Example : - Raj completed his test in 30 minutes, while Rajan took an entire hour to complete his test. Find the ratio of their time taken to complete the test.
Solution : - Time taken by Raj = 30 minutes

Time taken by Rajan = 1 hour = 60 minutes (Converting hours to minutes)

Required ratio = $\frac{Time\ \ taken\ \ by\ Raj}{Time\ \ taken\ \ by\ Rajan} = \frac{30}{60}$

$\frac{Time\ \ taken\ \ by\ Raj}{Time\ \ taken\ \ by\ Rajan} = \frac{1}{2}$

Time taken by Raj : Time taken by Ranjan = 1 : 2

So, the quantities being compared have to be converted to the same unit before comparison. In both methods, the ratio is time taken by Raj time taken by Rajan. If we want the ratio of time taken by

Rajan to time taken by Raj, it will be 1 : 2.

That is, time taken by Rajan : time taken by Raj = 1:2

Equivalent Ratios : -
In the above example, we can see that the ratio does not change when its first and second terms are multiplied or divided by the same number (non-zero). Such ratios are called equivalent ratios.
(The order of terms matter)

$\frac{5}{2} = \frac{5 \times 6}{2 \times 6} = \frac{30}{12}$

$\frac{21}{36} = \frac{21 \div 3}{36 \div 3} = \frac{7}{12}$

Thus, 30: 12 is an equivalent ratio of 5: 2 (obtained by multiplication) and
7:12 is an equivalent ratio of 21: 36 (obtained by division).

Ratio in the Simplest Form: - A ratio is said to be in the lowest or the simplest form when the HCF of its antecedent and consequent is 1.
In general, x:y is in the simplest form if the HCF of x and y is1.
To convert a ratio to the lowest form, we divide both the terms by their HCF.

Example : - Convert 75 : 100 to its simplest form.
Solution: HCF of 75 and 100 is 25

So, 75÷25 and 100 ÷ 25 gives 3 and 4, respectively.
75: 100 = 3: 4

Comparison of Ratios : -

Step 1: Express each ratio as a fraction in the simplest form.

Step 2: Make the denominators of the fractions the same by taking LCM and compare the numerators.

Example :- Which is greater? 8: 6 or 4: 10

Solution: 8 : 6 = $\frac{8}{6} = \frac{4}{3}$ and 4 : 10 = $\frac{4}{10} = \frac{2}{5}$

Now, LCM of 3 and 5 is 15.

Equating the denominators, we get

$\frac{4}{3} = \frac{4 \times 5}{3 \times 5} = \frac{20}{15}$ and $\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$

Now, compare the numerators.

20 > 6

Thus $\frac{20}{15} > \frac{6}{15}$

Therefore, 8: 6 > 4: 10.

QUESTION DISCUSSION

CHAPTER: RATIO AND PROPORTION

EXERCISE : 9.1

1. Express as a ratio:

(a) 10 p to 21 p (b) 500 to 421 (c) 17L to 52 L

(d) 6 g to 19 g (e) 12 m to 139 m

Solution : -

(b) 500 to 421

Required ratio = 500 : 421

(e) 12 m to 139 m

Required ratio = 12 : 139

2. Write the antecedent and consequent for all the parts of Q1.

Solution : -

(e) 12 m to 139 m

Required ratio = 12 : 139

Antecedent = 12 and Consequent = 139

3. Write 3 equivalent ratios for

(a) 1:7 (b) 15:6 (c) 91: 39 (d) 42:9 (e) 14:21

Solution :-

Three equivalent ratio of 15:6 are – 30 : 12 , 45 : 18 and 60 : 24

Solution:-

(c) 91 : 39

three equivalent ratio of 91 : 13 are -

7 : 1 , 14 : 2 and 21 : 3

910 : 130 , 9100 : 1300 and 91000 : 13000

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4. Express the ratio of the following in the simplest form:

(a) 150 mL to 1 L (b) 25 p to ₹ 10 (c) 120 g to 12 kg

(d) 250 m to 2 km (e) A decade to 1 year (f) A fortnight to a month

Solution : -

(a) 150 mL to 1 L

= 150 mL : 1000 mL

= 150 : 1000

= 3 : 20

5. Insert >, < or = in the □

(a) 5:6 □ 7:4 (b) 11:4 □ 121: 8 (c) 12: 18 □ 14:32

(d) 3:7 □ 7:3 (e) 1:3 □ 2:6 (f) 18:15 □ 16:36

Solution : -

(a) 5:6 < 7:4 (b) 11:4 <121: 8 (c) 12: 18 > 14:32

(d) 3:7 < 7:3 (e) 1:3 = 2:6 (f) 18:15 > 16:36

6. Divide 830 in the ratio 7: 3.

Solution : -

Let the numbers are 7x and 3x

A/Q, 7x + 3x = 830

Or, 10 x = 830

Or, x = 830/10

Or, x = 83

The numbers are 7 × 83 = 581 and 3 × 83 = 249

7. Three numbers are in the ratio 1 : 2 : 3. If their sum is 690, find the numbers.

8. The ratio of boys to girls in a primary school is 3 : 7. Find the number of boys and girls if the total number of students is 800.

9. The ratio of the length of a rectangle to its breadth is 3 : 2. If its perimeter is 130 m, find its length and breadth.

Solution:

Let length = 3x and breadth = 2x

Perimeter of Rectangle = 2 (Length + Breadth)

A/Q, 2 (3x + 2x) = 130

or , 10 x = 130

Or, x = 130/10

Or, x = 13

Length = 3 × 13 m = 39 m

Breadth = 2 × 13 m = 26 m

9/12/20

10. Preeti walked 3 km in 40 minutes and Raunak walked 4 km in 50 minutes. Find the ratio of their walking speed.

Solution : -

Speed = distance / Time

Speed of Preeti = 3 km / 40 min

Speed of Raunak = 4 km / 50 min

Speed of Preeti : Speed of Raunak = 3/40 : 4/50

11. 3 out of every 20 books in a library were in tatters. If the library had 1200 books, find the number of books in good condition.

12. The ratio of fruit pulp to sugar syrup in a squash concentrate is 5:3. How many litres of sugar syrup would 88 L of squash contain?

13. Riya spends three-fifths of her salary. If her salary is ₹ 5,00,000 find the

(a) ratio of savings to salary.

(c) amount she saved.

(b) savings to expenditure.

(d) amount she spent.

14. Divide ₹ 4255 among Ashwin, Sargam and Simran in the ratio $\frac{1}{4}:\frac{1}{5}:\frac{1}{6}$

Solution :-

Let Ashwin's share = $\frac{x}{4}$

Sargam's share = $\frac{x}{5}$

Simran's share = $\frac{x}{6}$

A/Q, $\frac{x}{4}+\frac{x}{5}+\frac{x}{6} = 4255$

=> $\frac{15x+12x+10x}{60} = 4255$

=> $\frac{37x}{60} = 4255$

=> $37x = 4255 \times 60$

=> $x = \frac{4255 \times 60}{37}$

=> $x = 115 \times 60 = 6900$

Therefore

Ashwin's share = ₹ $\frac{6900}{4}$ = ₹1725

Sargam's share =₹ $\frac{6900}{5}$ = ₹ 1380

Simran's share = ₹ $\frac{6900}{6}$ = ₹1150

Word Problems :

PROPORTION: - Two ratios are said to be in proportion when they are equal. The symbol for proportion is “ : : ” read as 'is as' or =" equal to.

In general, the numbers x, y, a and b are said to be in proportion if x: y:: a :b
or x: y =a:b where x is the first term, y is the second, a is the third and b is the fourth term.

The first term and the fourth term are called the **extremes,** while the second and the third terms are called **means** or middle terms.

In a proportion x : y :: a : b, bx = ay (Product of extremes = Product of means)

Example :

Are the ratios 3: 5 and 2: 3 in proportion?

Product of extremes = 3 x 3 =9

Product of means = 5 x 2 = 10

Since 9 # 10, 3 : 5 and 2: 3 are not in proportion.

CONTINUED PROPORTION: -

Three numbers are said to be in continued proportion when the ratio between two successive terms is the same.

Consider x, y, z such that x : y = y : z, then x, y and z are said to be in continued proportion.

The second term y' is said to be the mean proportion between the first term (x) and the third term (z). The third term is called the third proportion to the first and second terms.

Example : - Find the mean proportion between 12 and 3.

Solution: Let x be the mean proportion.

12 : x :: x : 3

Product of extremes = Product of means

12 × 3 = x × x

x × x = 36

Or, x × x = 6 x 6

x= 6

WORD PROBLEMAS: -

UNITARY METHOD

The method of finding first the value of one article from the value of a given numbers of articles and then finding the value of the required number of articles is called the **unitary method.**

Value of one article= (Value of given number of articles)/(*Number of articles)*

This method is used to find the value of one unit.

Example :- The cost of 5 bags is ₹ 250. Find the cost of 1 bag.

Solution:

Cost of 5 bags = ₹ 250

Cost of 1 bag = ₹ 250

Cost of 1 bag is = ₹ $\frac{250}{5}$ = ₹ 50

Example :- Pornima uses 12 L of oil in 3 months. How much oil does she use in 5 months?

Solution: Oil used in 3 months 12 L

Oil used in 1 month = 12 ÷ 3 = 4L

So, Oil used in 5 months = 4 × 5 = 20 L

6. Find the mean proportion between 20 and 5.

Solution:

Let x be the mean proportion.

20 : x :: x : 5

Product of extremes = Product of means

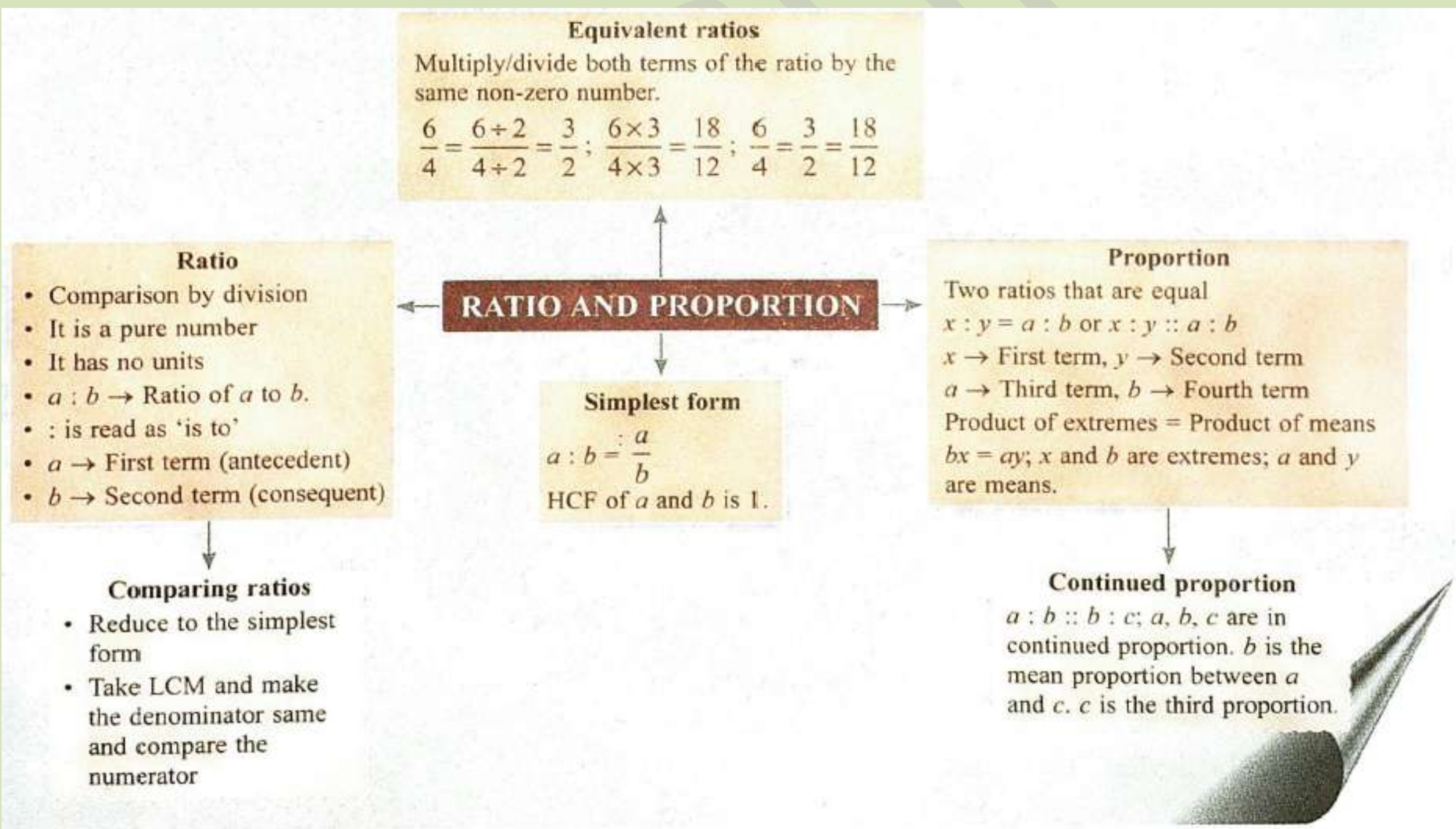
20 × 5 = x × x

x × x = 100

Or, x × x = 10 x 10

x= 10

Therefore mean proportion = 10



What is Unitary Method?

The unitary method is a method in which you find the value of a unit and then the value of a required number of units. What can units and values be?

Suppose you go to the market to purchase 6 apples. The shopkeeper tells you that he is selling 10 apples for Rs 100. In this case, the apples are the units, and the cost of the apples is the value. While solving a problem using the unitary method, it is important to recognize the units and values.

For simplification, always write the things to be calculated on the right-hand side and things known on the left-hand side. In the above problem, we know the amount of the number of apples and the value of the apples is unknown. It should be noted that the concept of ratio and proportion is used for problems related to this method.

Example of Unitary Method

Consider another example; a car runs 150 km on 15 litres of fuel, how many kilometres will it run on 10 litres of fuel?

In the above question, try and identify units (known) and values (unknown).

Kilometre = Unknown (Right Hand Side)

No of litres of fuel = Known (Left Hand Side)

Now we will try and solve this problem.

15 litres = 150 km

1 litre = 150/15 = 10 km

10 litres = 10 x 10 = 100 km

The car will run 100 kilometres on 10 litres of fuel.

Unitary Method in Ratio and Proportion

If we need to find the ratio of one quantity with respect to another quantity, then we need to use the unitary method. Let us understand with the help of examples.

Example: Income of Amir is Rs 12000 per month, and that of Amit is Rs 191520 per annum. If the monthly expenditure of each of them is Rs 9960 per month, find the ratio of their savings.

Solution:

Savings of Amir per month = Rs (12000 – 9960) = Rs 2040

In 12 month Amit earn = Rs.191520

Income of Amit per month = Rs 191520/12 = Rs. 15960

Savings of Amit per month = Rs (15960 – 9960) = Rs 6000

Therefore, the ratio of savings of Amir and Amit = 2040:6000 = 17:50