FRACTIONS
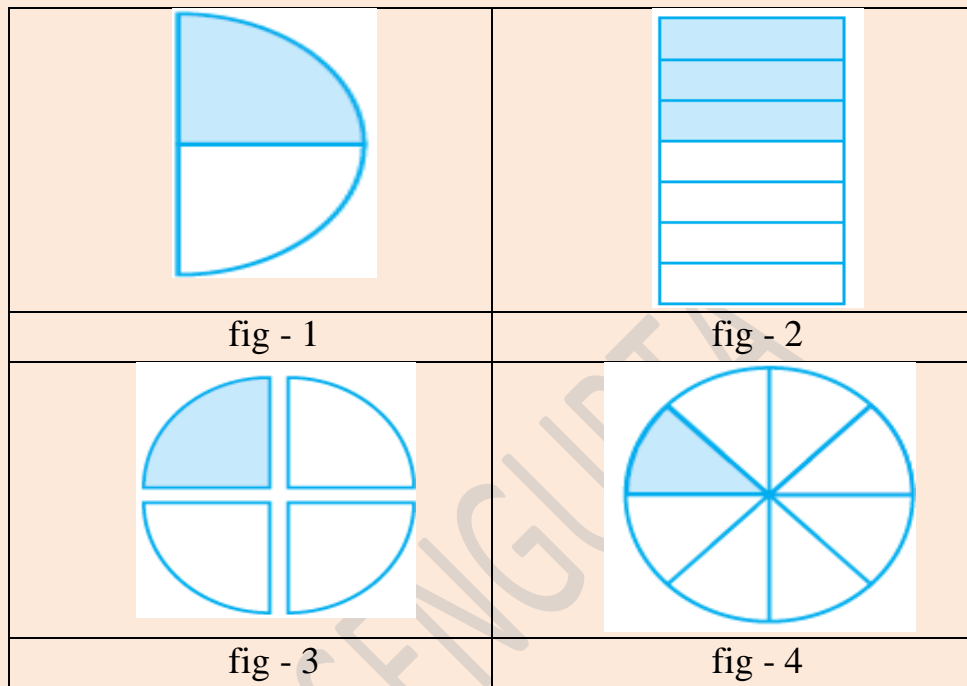
1. <mark>Introduction of Fraction</mark> - The word fraction originally comes from the Latin ward fractio which means , breaking into pieces.

A fraction means a part of a collection or a whole. The whole may be a single object or a group of objects.

Look at the following fractions -

fig - 1	fig - 2
fig - 3	fig - 4

In fig – 1, out of total two parts one part is shaded , so the corresponding fraction is $\frac{1}{2}$.

In fig – 2, out of total seven parts three parts is shaded , so the corresponding fraction is $\frac{3}{7}$.

In fig – 3, out of total four parts one part is shaded , so the corresponding fraction is $\frac{1}{4}$.

In fig – 4, out of total eight parts one part is shaded , so the corresponding fraction is $\frac{1}{8}$.

In a fraction $\frac{a}{b}$, a is known as numerator of the fraction and b is known as denominator of the fraction.

2. **Representation of fraction on the Number line** : -

Example : Represent $\frac{6}{7}$ on the number line.

Steps : Draw a Number line and mark 8 points on it to divide the number line in 7 parts as shown.

Mark first point as $\frac{0}{7}$ ($= 0$), second point as $\frac{1}{7}$, third point as $\frac{2}{7}$ and so on , here the eight point is $\frac{7}{7} = 1$.
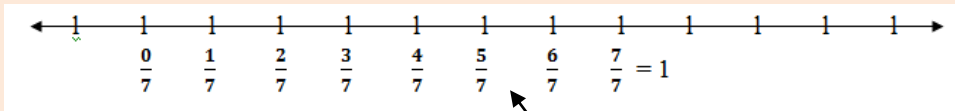
The required point $\frac{6}{7}$ is our seventh point on the number line.

3. TYPES OF FRACTIONS

(i)   Proper fraction: A fraction whose numerator is less than its denominator is known as a Proper Fraction.

In the fraction $\frac{a}{b}$, if a < b, then it is called a proper fraction.

For example - $\frac{3}{5}, \frac{7}{10}, \frac{21}{37}$ are all Proper fractions.

(ii)   Improper fraction: A fraction whose numerator is greater than its denominator is known as an Improper Fraction.

In the fraction $\frac{a}{b}$, if a > b, then it called Improper fraction.

For example - $\frac{13}{5}, \frac{7}{5}, \frac{13}{7}$ are all Improper fractions.

(iii)   Mixed Fraction : A mixed fraction is an improper fraction when expressed as a combination of a whole number and a proper fraction.

A mixed fraction is expressed as $b\frac{c}{d}$, where b , c , d are whole numbers (other than 0) and c < d.

For example : $3\frac{13}{15}, 2\frac{3}{5}, 6\frac{13}{17}$ are all Mixed fractions.

Unit fraction : Fraction with 1 in the numerator is known as Unit Fraction.

For example : $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{15}$, $\frac{1}{25}$ are all unit fractions.

(iv) Decimal fractions : Fractions having denominators as multiples of 10 are known as Decimal fractions

For example : $\frac{1}{10}$, $\frac{6}{100}$, $\frac{23}{1000}$ are all Decimal fractions.

(v) Vulgar Fractions : Fractions having denominator as natural numbers other than 10, 100, 1000, 10000, ... etc are called as Vulgar Fractions.

For example : $\frac{6}{7}$, $\frac{15}{123}$, $\frac{100}{267}$,... are all Vulgar Fractions.

(vi) Irreducible Fractions : Fractions in which the numerator and the denominator are integers but the numerator and denominator have no common integers except 1.

For example : $\frac{1}{6}$, $\frac{2}{3}$ and $\frac{7}{9}$ are Irreducible Fractions.

4. Equivalent fractions :   Two or more fractions which represent the same part of a whole number are called equivalent fraction.

For example : $\frac{5}{7}$, $\frac{10}{14}$, $\frac{15}{21}$, $\frac{20}{28}$, $\frac{25}{35}$, are all equivalent fractions of $\frac{5}{7}$.

(To get an equivalent fraction of a given fraction , we multiply or divide its numerator and denominator with the same non zero number.)

5. Like fractions: The fractions with the same denominator are known as Like fractions .

For example : $\frac{5}{13}$, $\frac{10}{13}$, $\frac{15}{13}$, $\frac{7}{13}$, are all like fractions.

Unlike fractions: The fractions with different denominators are known as Unlike fractions.

For example : $\frac{5}{13}$, $\frac{10}{11}$, $\frac{15}{19}$, $\frac{17}{23}$, are all unlike fractions.

Simplest (Lowest) form of a fraction : A fraction is said to be in simplest form if its numerator and denominator have no common factor except 1.

For example : Simplest form of $\frac{144}{240}$ is $\frac{3}{5}$.

6. <mark>Comparison of Fractions</mark> :   Two or more fractions can be compare in the following ways –

(a)   Making Denominator Same : When denominators of two or more fractions are same (equal) , then the fraction with greater numerator is greater .

For example : (i) Let $\frac{2}{5}$ and $\frac{3}{5}$, Here denominator of both fractions are same , so the fraction with greater numerator is greater , since $3 > 2$, so $\frac{3}{5} > \frac{2}{5}$.

(b)   Comparison by Cross Multiplication :   Let $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions , then by cross multiplication we have , $a \times d$ and

$b \times c$, now if

(i) $a \times d > b \times c$, then $\frac{a}{b} > \frac{c}{d}$

(ii) $a \times d < b \times c$, then $\frac{a}{b} < \frac{c}{d}$

(c)   Every proper fraction is less than any improper fractions - $\frac{3}{7}$ is a proper fraction and $\frac{19}{13}$ is an improper fraction , so $\frac{3}{7} < \frac{19}{13}$.

7. **<mark>Ascending and Descending order</mark>**

      (a)   Ascending order: To express from smaller to greater

Example : Arrange the following fractions in ascending order :

$\frac{7}{8}$, $\frac{13}{16}$, $\frac{15}{12}$

Solution : Here LCM of 8, 16 and 12 is 48,

Now we will convert all three fractions $\frac{7}{8}$, $\frac{13}{16}$ and $\frac{15}{12}$ to their respective equivalent fractions with denominator 48 as –

$\frac{7}{8} = \frac{7 \times 6}{8 \times 6} = \frac{42}{48}$, $\frac{13}{16} = \frac{13 \times 3}{16 \times 3} = \frac{39}{48}$

$\frac{15}{12} = \frac{15 \times 4}{12 \times 4} = \frac{60}{48}$

Now, $39 < 42 < 60$

Required ascending order is $\frac{13}{16} < \frac{7}{8} < \frac{15}{12}$

(b) Descending order : To express from greater to smaller.

Example: Arrange the following fractions in descending order:

$\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{8}$, $\frac{5}{9}$

Solution: Here LCM of 3, 5, 8 and 9 $= 360$.

Now we will convert all the four fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{8}$ and $\frac{5}{9}$ to their respective equivalent fractions with denominator 360 as –

$\frac{2}{3} = \frac{2 \times 120}{3 \times 120} = \frac{240}{360}$, $\frac{4}{5} = \frac{4 \times 72}{5 \times 72} = \frac{288}{360}$

$$\frac{1}{8} = \frac{1 \times 45}{8 \times 45} = \frac{45}{360} \ , \quad \frac{5}{9} = \frac{5 \times 40}{9 \times 40} = \frac{200}{360}$$

Here $288 > 240 > 200 > 45,$

The required descending order is $\dfrac{4}{5} > \dfrac{2}{3} > \dfrac{5}{9} > \dfrac{1}{8}$

8. <mark>Addition and Subtraction of fractions</mark>:

(i) When we have to add two or more fractions, first we make their denominator same by taking LCM of denominators,

For example: Add $\dfrac{3}{4}$, $\dfrac{5}{8}$ and $\dfrac{19}{10}$,   (Here LCM of 4, 8 and 10 $= 40$)

$$\frac{3}{4} + \frac{5}{8} + \frac{19}{10}$$
$$= \frac{30}{40} + \frac{25}{40} + \frac{76}{40}$$
$$= \frac{30 + 25 + 76}{40}$$
$$= \frac{131}{40} = 3\frac{11}{40}$$

(ii) When we have to subtract one fraction from another fraction , first we make their denominator same by taking LCM of denominators,

For example: Subtract $\dfrac{13}{20}$ from $\dfrac{11}{5}$

$$= \frac{11}{5} - \frac{13}{20} \quad (\text{ Here LCM of 5 and 20} = 20)$$
$$= \frac{44}{20} - \frac{13}{20}$$
$$= \frac{44 - 13}{20}$$
$$= \frac{31}{20} = 1\frac{11}{20}$$

9. <mark>MULTIPLICATION OF FRACTIONS:</mark>

(i) Multiplication of a Fraction by a whole number : If the fraction is a proper fraction or improper fraction by a whole number , then simply we have to multiply the numerator of the fraction by that whole number.

If the fraction is a mixed fraction first we have to convert it in to improper fraction and then multiply its numerator by the given whole number.

For example:

(a) $\dfrac{12}{17} \times 5 = \dfrac{12 \times 5}{17} = \dfrac{60}{17} = 3\dfrac{9}{17}$

(b) $\dfrac{12}{5} \times 3 = \dfrac{12 \times 3}{5} = \dfrac{36}{5} = 7\dfrac{1}{5}$

(c) $6\dfrac{1}{5} \times 4 = \dfrac{31}{5} \times 4 = \dfrac{31 \times 4}{5} = \dfrac{124}{5} = 24\dfrac{4}{5}$

(ii) **Multiplication of a Fraction by a Fraction** : To multiply a fraction by another fraction we have to multiply numerators and denominators of both the fractions separately.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions then the product of $\frac{a}{b}$ and $\frac{c}{d}$ $= \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

For example :

(a) Multiply $\frac{3}{5}$ by $\frac{7}{12}$ $= \frac{3}{5} \times \frac{7}{12} = \frac{3 \times 7}{5 \times 12} = \frac{21}{60} = \frac{7}{20}$

(b) Multiply $\frac{12}{13}$ by $\frac{8}{12}$ $= \frac{12}{13} \times \frac{8}{12} = \frac{12 \times 8}{13 \times 12} = \frac{96}{156} = \frac{8}{13}$

(c) Multiply $2\frac{3}{5}$ by $\frac{6}{7}$ $= \frac{13}{5} \times \frac{6}{7} = \frac{78}{35} = 2\frac{8}{35}$

(iii) Fraction as an operator 'of' :   Of means multiply ,

$\frac{1}{5}$ of 20 , means $\frac{1}{5} \times 20 = \frac{1 \times 20}{5} = \frac{20}{5} = \frac{20}{5} = 4$

10. Reciprocal of a Fraction : Reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$.

To find the reciprocal of any fraction simply we have to interchange their numerator and denominator.

For example : Reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$

11. Division of Fraction -

(i) Division of a Fraction by a Whole number: To divide a fraction $\frac{a}{b}$ by a whole number c, we have to multiply $\frac{a}{b}$ by the reciprocal of c $= \frac{1}{c}$

That is, $\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c} = \frac{a \times 1}{b \times c} = \frac{a}{bc}$

Foe example:

Divide $\frac{13}{5}$ by 4 $= \frac{13}{5} \div 4 = \frac{13}{5} \times \frac{1}{4} = \frac{13 \times 1}{5 \times 4} = \frac{13}{20}$

(ii)   Division of a Fraction by a Fraction: To divide a fraction by another fraction , we have to multiply the first fraction by the reciprocal of the second fraction.

To divide $\frac{a}{b}$ by another fraction $\frac{c}{d}$, we have to multiply $\frac{a}{b}$ by the reciprocal of $\frac{c}{d}$, that is

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c} = \frac{ad}{bc}$

For example :

(a) Divide $\frac{3}{5}$ by $\frac{7}{12}$

$= \frac{3}{5} \div \frac{7}{12} = \frac{3}{5} \times \frac{12}{7} = \frac{3 \times 12}{5 \times 7} = \frac{36}{35} = 1\frac{1}{35}$

(b) Divide $2\frac{2}{5}$ by $\frac{8}{12}$

$= \frac{12}{5} \div \frac{8}{12} = \frac{12}{5} \times \frac{12}{8} = \frac{12 \times 12}{5 \times 8} = \frac{18}{5} = 3\frac{3}{5}$

(c) Divide $2\frac{3}{7}$ by $3\frac{2}{5}$

$= \frac{17}{7} \div \frac{17}{5} = \frac{17}{7} \times \frac{5}{17} = \frac{5}{7}$