SUBJECT : SCIENCE (PHY)

CHAPTER-12:
ELECTRICITY

TOPIC-5:

COMBINATION OF RESISTORS
OR
(GROUPING OF RESISTORS)

OBJECTIVES:

Upon completion of the topic, you will be able to

1. DEFINE SERIES COMBINATION AND PARALLEL COMBINATION OF RESISTORS
2. DEFINE EQUIVALENT RESISTANCE
3. DRAW DIAGRAM TO SHOW SERIES & PARALLEL COMBINATION OF RESISTANCES
4. WRITE THE DIFFERENCES BETWEEN SERIES AND PARALLEL COMBINATION
5. SOLVE NUMERICAL BASED ON SERIES AND PARALLEL COMBINATION OF RESISTANCES

Combination of Resistors

- Resistors can be combined in two ways –

i. In series.

ii. In parallel.

Resistors in Series

- When two (or more) resistors are connected end to end consecutively, they are said to be connected in series.
- According to the law of combination of resistance in series: **The combined resistance of any number of resistances connected in series is equal to the sum of the individual resistances.**

$$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots$$

Equivalent resistance: The total resistance (effective resistance) between two points is called Equivalent resistance (R)

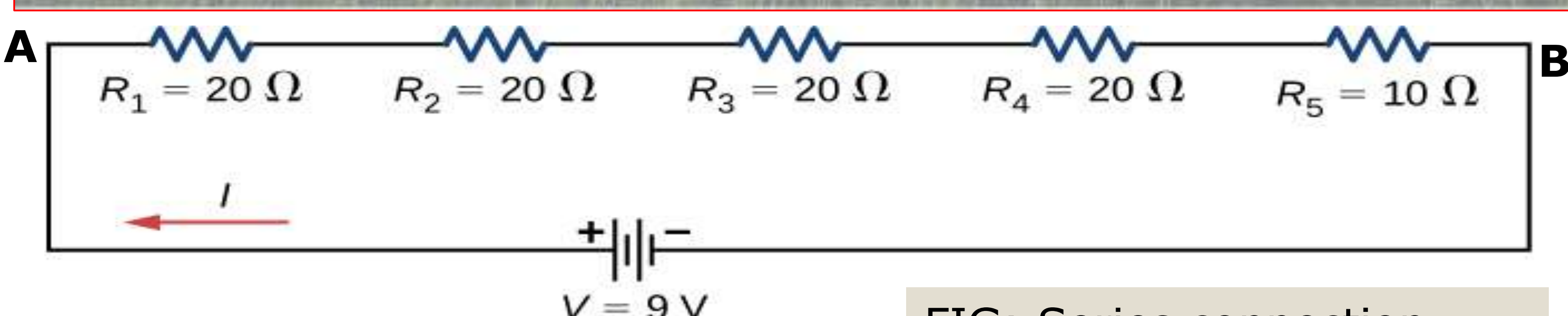


FIG: Series connection

I. When a number of resistors connected in series are joined to the terminal of a battery, then each resistance has a different potential difference across its ends (which depends on the value of resistance). But the total potential difference across all the ends of all the resistors in series is equal.

II. When a number of resistors are connected in series, then the same current flows through each resistance.

Resultant of Resistances connected in Series

- The figure shows three resistances R_1, R_2, R_3 connected in series. Now suppose potential difference across resistance R_1 is V_1, R_2 is V_2 and R_3 is V_3. Let potential difference across battery be V, then :

$$V = V_1 + V_2 + V_3.$$

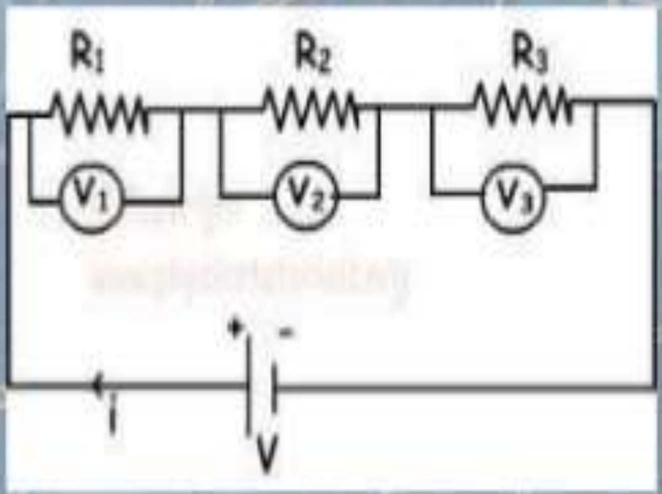
Applying Ohm's law to the whole circuit : $V = IR$. ....(1)

Applying Ohm's law to the three resistors separately, we get:

$V_1 = I \times R_1$. .... (2)

$V_2 = I \times R_2$. .... (3)

$V_3 = I \times R_3$. .... (4)



Substituting (2), (3), (4) in (1)

$IR = IR_1 + IR_2 + IR_3$

OR, $IR = I(R_1 + R_2 + R_3)$

Or, $R = R_1 + R_2 + R_3$.

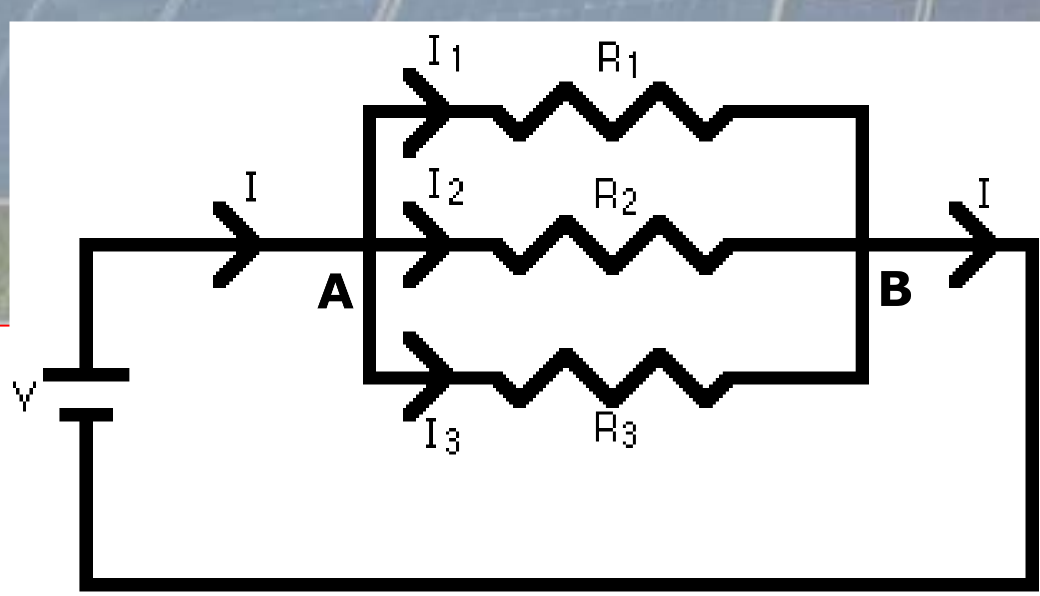
Therefore we conclude that the sum total resistance in a series resistance connection is equal to the sum of all the resistances.

Resistors in Parallel

- When two (or more) resistors are connected between the same points, they are said to be connected in parallel.
- According to the law of combination of resistance in parallel: **The reciprocal of the combined resistance of any number of resistances connected in parallel is equal to the sum of the reciprocals of the individual resistances.**

$$1/R = 1/R_1 + 1/R_2 + 1/R_3 + \ldots\ldots\ldots$$

- When a number of resistances are connected in parallel then their combined resistance is less than the smallest individual resistance.



- *When a number of resistance are connected in parallel, then the potential difference across each resistance is same which is equal to the voltage of battery applied.*
- *When a number of resistances connected in parallel are joined to the two terminals of a battery, then different amounts of current flow through each resistance (which depend on the value of resistance). But the current flowing through each parallel resistance, taken together, is equal to the current flowing in the circuit as a whole. Thus, when a number of resistance are connected in parallel, then the sum of current flowing through all the resistances is equal to the total current flowing in the circuit.*

Resultant of Resistances connected in Parallel

- The figure shows three resistances R_1,R_2,R_3 connected in series. Now suppose currant across resistance R_1 is I_1, R_2 is I_2 and R_3 is I_3. Let total current in the circuit be I, then:

$$I = I_1+I_2+I_3.$$

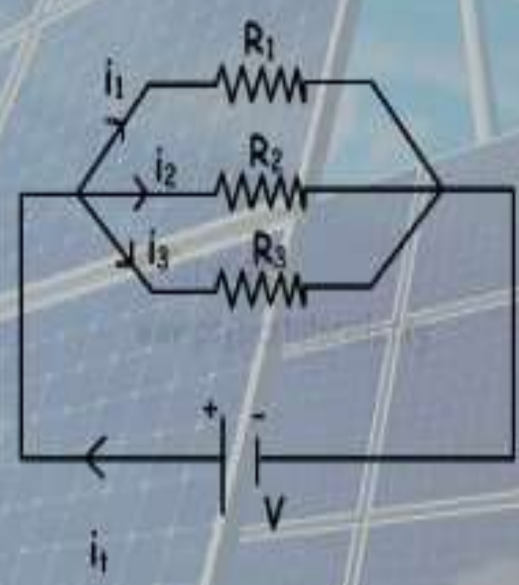
Applying Ohm's law to the whole circuit : I = V/R. ....(1)

Applying Ohm's law to the three resistors separately, we get:

$I_1 = V / R_1$. .... (2)

$I_2 = V / R_2$. .... (3)

$I_3 = V / R_3$. .... (4)



Substituting (2), (3), (4) in (1)

$V/R = V/R_1 + V/R_2 + V/R_3$

OR, $V/R = I\,(1/R_1 + 1/R_2 + 1/R_3)$

Or, $1/R = 1/R_1+1/R_2+1/R_3$.

Therefore we conclude that the sum total resistance in a parallel resistance connection is equal to the sum of reciprocal of all the resistances.

Parallel and Series connection

Parallel connection

- If one electric appliance stops working due to some defect, then all other appliances keep working normally.
- In parallel circuits, each electric appliance has its own switch due to which it can be turned on or off independently.
- Each appliance gets same voltage as that of power source.
- Overall resistance of household circuit is reduced due to which the current from power supply is high.

Series connection

- If one electric appliance stop working due to some defect, then all other appliances stop working.
- All the electric appliances have only one switch due to which they cannot be turned on or off separately.
- In series circuit, the appliances do not get same voltage (220 V) as that of the power supply line.
- In series circuit the overall resistance of the circuit increases due to which the current from the power source is low.

QUESTIONS AND NUMERICAL

Example

If 2 Resistors - 3 Ω and 4 Ω are connected in parallel. What is the resistance of the circuit

Since resistors are connected in parallel,

Resistance of Circuit is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4}$$

$$\frac{1}{R} = \frac{4 + 3}{3 \times 4}$$

$$\frac{1}{R} = \frac{7}{12}$$

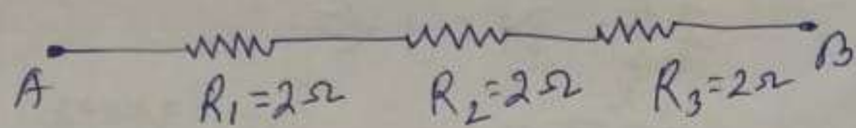
$$R = \frac{12}{7}$$

R = **1.71 Ω**

QUESTIONS

Question: Find equivalent resistance

(1)

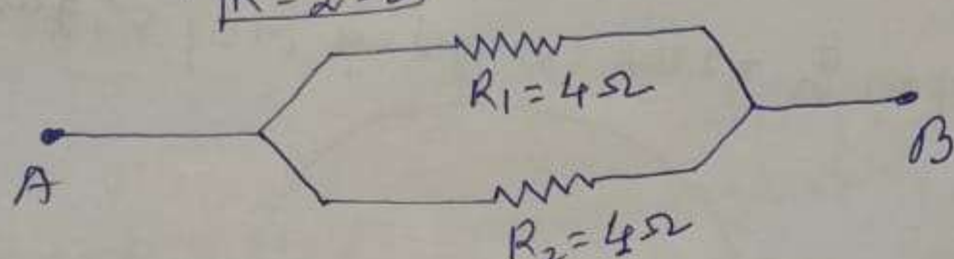


Sol^n: $R = R_1 + R_2 + R_3 =$

$= 2 + 2 + 2 = 6\Omega$

$\therefore$ $\boxed{R = 2\Omega}$

(2)



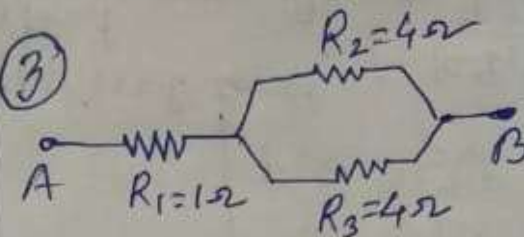
Sol^n: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

$\therefore$ $\boxed{R = 2\Omega}$

(3)



Sol^n: Here, $R_2 \parallel R_3$

$\therefore \frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_3}$

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

$= \frac{1}{2}$

$\therefore R = 2\Omega$

Now, R_1 and R are in series.

$\therefore$ Equivalent Resistance

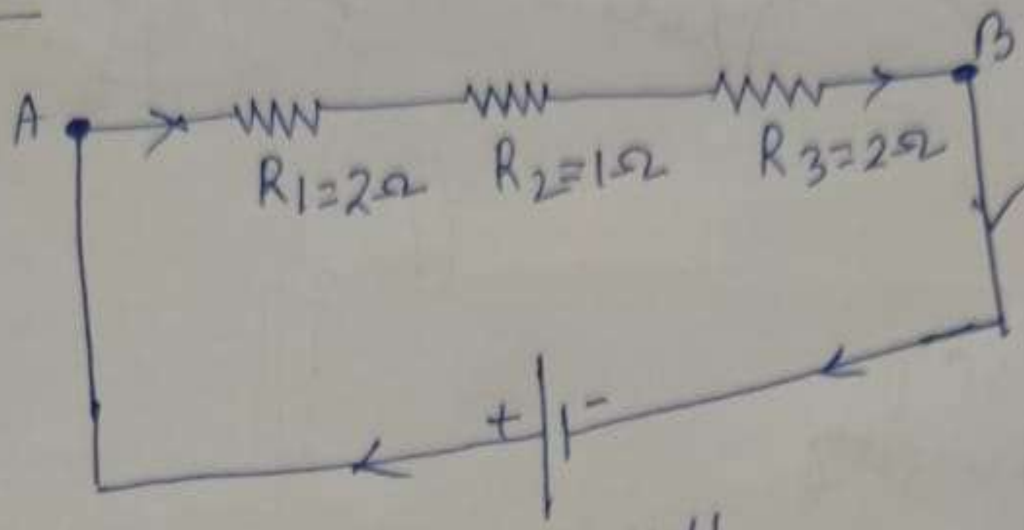
$R' = R_1 + R$

$= 1 + 2 = 3\Omega$

$\therefore$ $\boxed{R' = 3\Omega}$

QUESTIONS

Question:



(a) Find the equivalent resistance (effective resistance)

(b) Find Potential difference across R_1, R_2 & R_3

Soln: (a) Equivalent Resistance: $R = R_1 + R_2 + R_3 =$

$= 2 + 1 + 2 = 5\Omega$

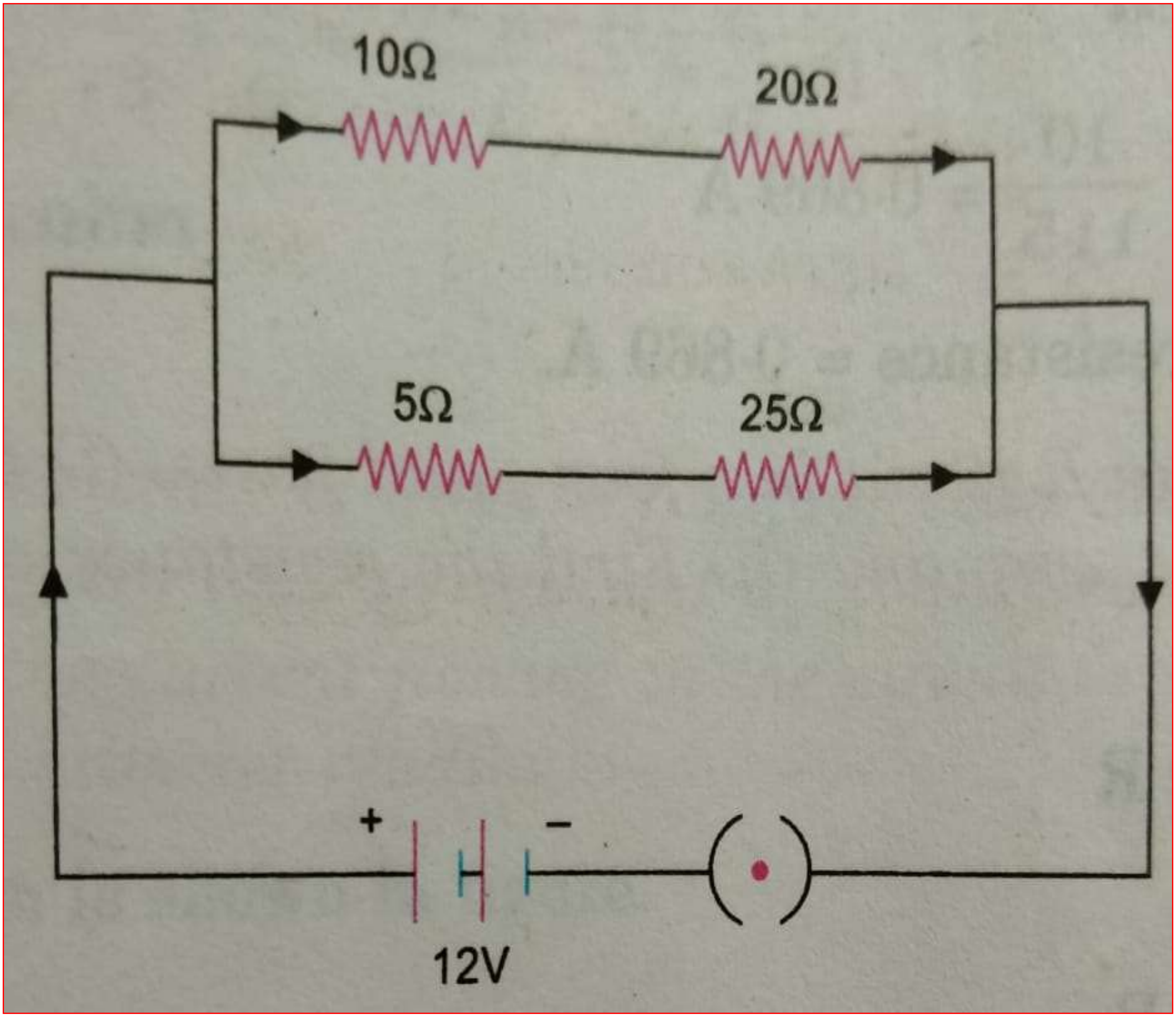
Current $I = \frac{V}{R} = \frac{10}{5} = 2A$.

(b) PD across R_1: $V_1 = IR_1 = 2 \times 2 = 4$ volt.

PD across R_2: $V_2 = IR_2 = 2 \times 1 = 2$ Volt

PD across R_3: $V_3 = IR_3 = 2 \times 2 = 4$ volt.

QUESTION: (a) Find total resistance in the circuit
(b) Find the total current in the circuit
(c) Find the potential difference across 10 ohm and 5 ohm.



a. 15 ohm
b. 0.8 ohm
c. 4 volt & 2 volt

ANSWER

IMPORTANT NOTES

1. To increase resistance, resistors are connected in series
2. To decrease resistance, resistors are connected in parallel
3. When a very high resistance is connected with a very low resistance in parallel, the total resistance become almost equal to the low resistance (for example 1 ohm & 1000 ohm when connected in parallel, the equivalent resistance is nearly equal to 1 ohm)
4. Show how would you join three resistors, each of resistance 9 W so that the equivalent resistance of the combination is (i) 13.5 ohm (ii) 6 ohm?

Hints: (i) To get equivalent resistance 13.5 ohm we need to connect the resistors as shown in fig-1(calculate & verify it).

(ii) To get equivalent resistance 6 ohm we need to connect the three resistors as shown in Fig-2 (calculate & verify it)

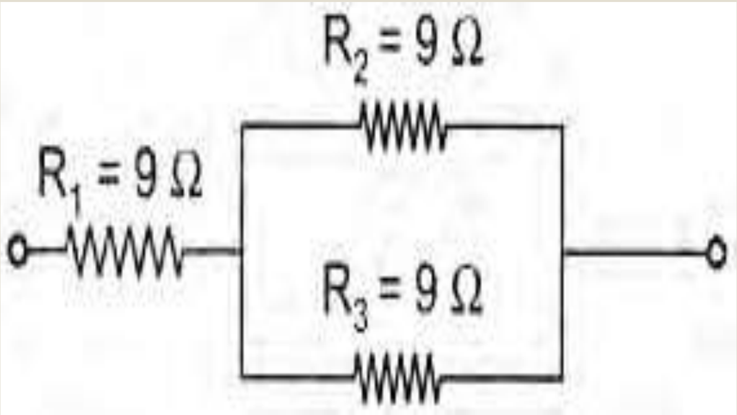


Fig : 1

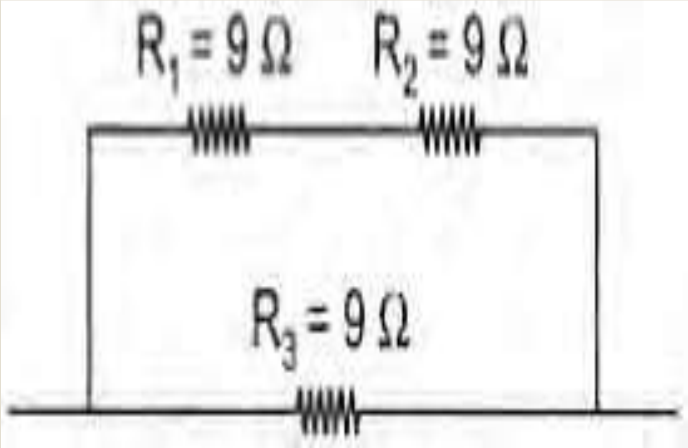


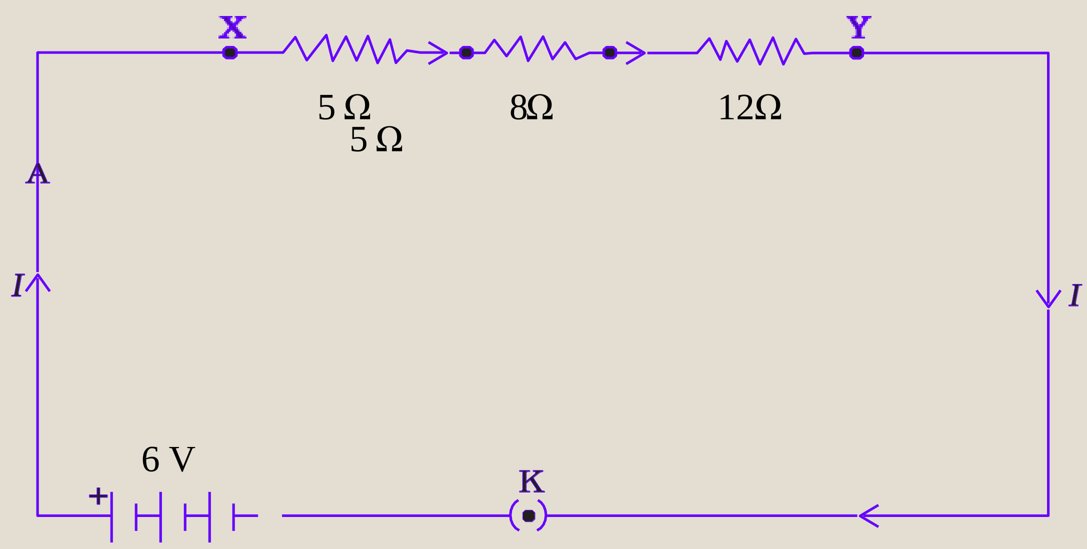
Fig : 2

Question 1:

Draw a schematic diagram of a circuit consisting of a battery of three cells of 2 V each, a 5 Ω resistor, an 8 Ω resistor, and a 12 Ω resistor, and a plug key, all connected in series.

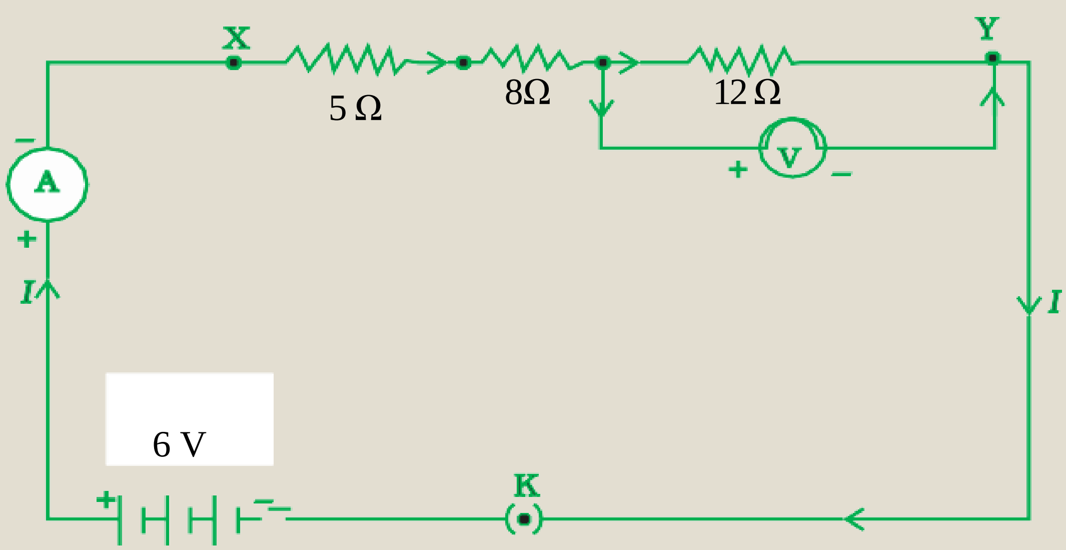
Answer 1:

The required schematic diagram is given below:



Question 2:

Redraw the circuit of Question 1, putting in an ammeter to measure the current through the resistors and a voltmeter to measure the potential difference across the 12 Ω resistor. What would be the readings in the ammeter and the voltmeter



Ammeter reading: I = V/R = 6/25= 0.24 A

Voltmeter reading across 12 ohm: V = IR= 0.24 X 12 = 2.88 Volt

Question 4:

An electric lamp of 100 Ω, a toaster of resistance 50 Ω, and a water filter of resistance 500 Ω are connected in parallel to a 220 V source. What is the resistance of an electric iron connected to the same source that takes as much current as all three appliances, and what is the current through it?

Answer 4:

Given that the electric lamp of 100 Ω, a toaster of resistance 50 Ω and water filter of resistance 500 Ω are connected in parallel.

The net resistance in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Here, $R_1 = 100\ \Omega$, $R_2 = 50\ \Omega$ and $R_3 = 500\ \Omega$

So,

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{50} + \frac{1}{500}$$

1/R = 1/10 + 1/50 + 1/500

$$= \frac{5 + 10 + 1}{500} = \frac{16}{500}$$

$$\Rightarrow R = \frac{500}{16} = 31.25\ \Omega$$

Now, using Ohm's law V = IR, we have

$$I = \frac{V}{R} = \frac{220\ V}{31.25\ \Omega} = 7.04\ A$$

Hence, the resistance of electric iron is 31.25 Ω and current through it is 7.04 A.

Question 5:

What are the advantages of connecting electrical devices in parallel with the battery instead of connecting them in series?

Answer 5:

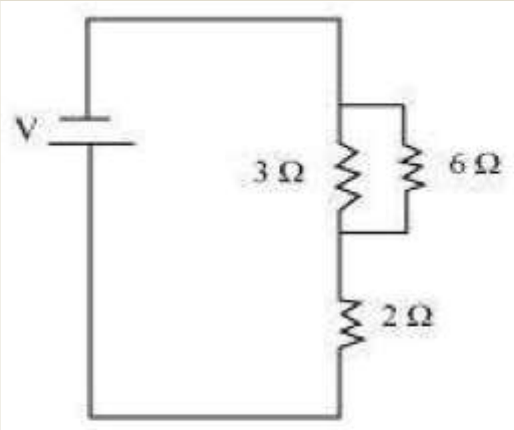
In parallel there is no division of voltage among the appliances. The potential difference across each appliance is equal to the supplied voltage and the total effective resistance of the circuit can be reduced by connecting electrical appliances in parallel.

Question 6:

How can three resistors of resistances 2 Ω, 3 Ω, and 6 Ω be connected to give a total resistance of **(a)** 4 Ω, **(b)** 1 Ω?

Answer 6:

(a). To get total resistance 4 Ω, connect 3 Ω and 6 Ω resistors in parallel and 2 Ω resistance in series with the resultant.



Since, 3 Ω and 6 Ω resistors in parallel, so the net resistance , 1/R = 1/3 + 1/6 = ½

So R = 2 ohm.

Now this resistance in in series with another 2 ohm resistor. Hence net resistance is

Rq = 2 + 2 = 4 ohm

(b) To get total resistance 1 ohm, we need to connect 2, 3 and 6 ohm resistors in parallel. 1/R = 1/2+ 1/3 + 1/6 = 6/6 = 1 . Thus R = 1 ohm

Question 7:

What is **(a)** the highest, **(b)** the lowest total resistance that can be secured by combinations of four coils of resistance 4 Ω, 8 Ω, 12 Ω, 24 Ω?

Answer 7:

Connecting resistors in series always gives maximum resistance and parallel gives minimum resistance.

(a). The highest total resistance is given by

$$R = R_1 + R_2 + R_3 + R_4 = 4\ \Omega + 8\ \Omega + 12\ \Omega + 24\ \Omega = 48\ \Omega$$

(b) Lowest total resistance is obtained by connecting resistors in parallel

$$1/R = 1/4 + 1/8 + 1/12 + 1/24 = 12/24 = 1/2$$

R = 2 ohm.

Question 8:

A battery of 9 V is connected in series with resistors of 0.2 Ω, 0.3 Ω, 0.4 Ω, 0.5 Ω and 12 Ω, respectively. How much current would flow through the 12 Ω resistor?

Answer 8:

Total resistance of resistors when connected in series is given by

$$R = R_1 + R_2 + R_3 + R_4 + R_5$$

$$\Rightarrow R = 0.2\ \Omega + 0.3\ \Omega + 0.4\ \Omega + 0.5\ \Omega + 12\ \Omega = 13.4\ \Omega$$

According to Ohm's law, V = IR

$$\Rightarrow I = \frac{V}{R} = \frac{9}{13.4} = 0.67\ A$$

There is no current division occurring in a series circuit. So, the current through the 12 Ω resistor will be same as 0.67 A.

Question 9:

How many 176 Ω resistors (in parallel) are required to carry 5 A on a 220 V line?

Answer 9

Let the total number of resistors be *x*.

Given that:

Current I = 5 A and Potential Difference V = 220 V

According to Ohm's law, V = IR , R = V/I = 220/5 = 44 ohm

Now for *x* number of resistors of resistance 176 Ω, the equivalent resistance of the resistors connected in parallel is 44 Ω.

$$\frac{1}{44} = \frac{1}{176} + \frac{1}{176} + \frac{1}{176} + \frac{1}{176} + ..x\,times$$

$$\Rightarrow \frac{1}{44} = \frac{x}{176}$$

$$\Rightarrow x = \frac{176}{44} = 4$$

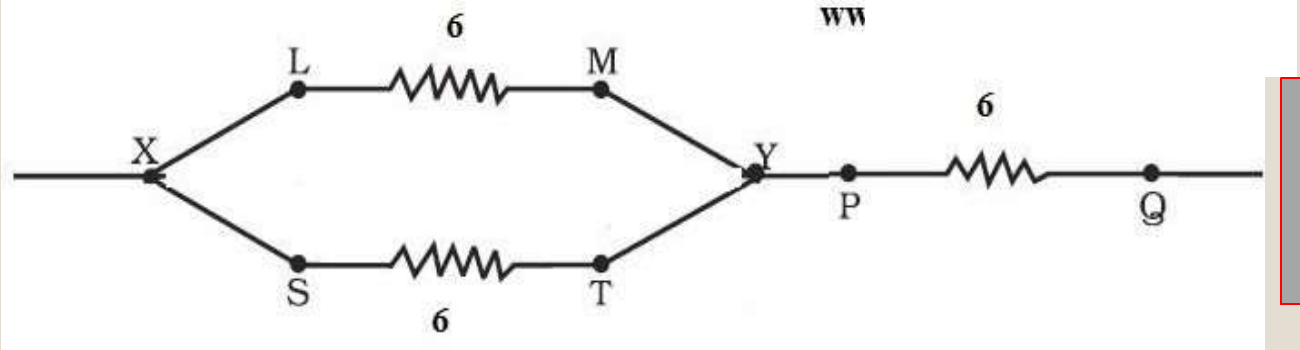
Therefore, 4 resistors of 176 Ω are required to draw the given amount of current

Question 10:

Show how you would connect three resistors, each of resistance 6 Ω, so that the combination has a resistance of (i) 9 Ω, (ii) 4 Ω.

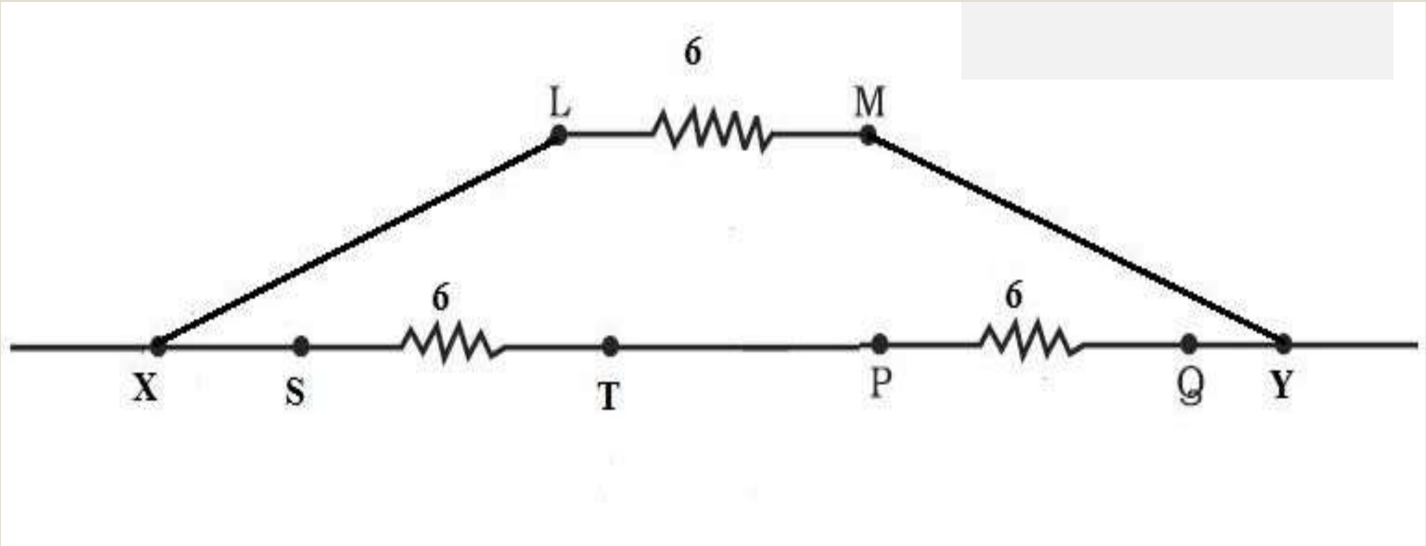
Answer 10:

(i). To get total 9 Ω resistance from three 6 Ω resistors, we should connect two resistors in parallel and the third resistor in series with the resultant. The combination is given as follows:



Equivalent of 6 & 6 ohm is = 3 ohm
Net resistance:
R = 3 + 6=9 ohm

(ii). To get total 4 Ω resistance from three 6 Ω resistors, we should connect two resistors in series and the third resistor in parallel with the resultant. The combination is given as follows:



Net resistance: 1/R = 1/6 + 1/12 = 3/12 = 1/4

R = 4 ohm

Question 11:

Explain the following.

a) Why is the tungsten used almost exclusively for filament of electric lamps?
b) Why are the conductors of electric heating devices, such as bread-toasters and electric irons, made of an alloy rather than a pure metal?
c) Why is the series arrangement not used for domestic circuits?
d) How does the resistance of a wire vary with its area of cross-section?
e) Why are copper and aluminium wires usually employed for electricity transmission?

Answer 11:

a) The melting point and resistivity of tungsten are very high. It does not burn readily at a high temperature. The electric lamps glow at very high temperatures. Hence, tungsten is mainly used as heating element of electric bulbs.

b) The conductors of electric heating devices such as bread toasters and electric irons are made of alloy because resistivity of an alloy is more than that of metals. It produces large amount of heat and do not burn easily.

c) There is voltage division in series circuits. Each component of a series circuit receives a small voltage for a large supply voltage. As a result, the amount of current decreases and the device becomes hot. Hence, series arrangement is not used in domestic circuits.

d) Resistance (*R*) of a wire is inversely proportional to its area of cross-section (A)

$$R \propto \frac{1}{A}$$

e) Copper and aluminium wires have low resistivity. They are good conductors of electricity. Hence, they are usually employed for electricity transmission.

Question 12: **(HOME- ASSIGNMENT)**

Find the equivalent resistance when the following are connected in parallel –

(a) 1 Ω and 10^6 Ω, **(b)** 1 Ω and 10^3 Ω, and 10^6 Ω.

Question-13:

what do you understand by series and parallel combination of resistors. Which kind of combination is generally used for connecting appliances in household wiring system and why?

Question-14:

write any three differences between series and parallel combination of resistors.

Question-15:

There is an electric circuit having an appliance of 1000 ohm resistance. How can the effective resistance in the circuit be reduced nearly 1 ohm without removing battery and other components existing in the circuit?

Question-16:

Two electric bulbs are connected in an electric circuit once in series and once in parallel across same potential difference. In which case the bulbs will glow brighter. Give reason to support your answer.

Question-17:

with increase of length resistance of a conductor increases and vice versa. Give reason.