Arithmetic Progressions

Sequence

- A set of numbers arranged in definite order according to some rule is called a sequence.
- E.g.
- (i) 4, 7, 10, 13, 16, …
- (ii) 2, 6, 18, 54, …
- (iii) 5, 3, 1, -1, -3, …
- are sequences because in each arrangement there is some rule according to which numbers are written.

- In (i) each term is obtained by adding 3 to the preceding term.
- In (ii) each term is obtained by multiplying the preceding term by 3
- In (iii) each term is obtained by subtracting 2 from the preceding term.

Series

- The algebraic sum of the terms of a sequence or a progression is called series.
- (i) 4 + 7 + 10 + …
- (ii) 2 + 6 + 18 + …
- (iii) 5 + 3 + 1 + …
- The sum of the first n terms of the series is denoted by S_n.
- The sum of first two terms is denoted by S_2,
- Sum of first seven terms is denoted by S_7 and so on.

Series

- Each term of sequence is denoted by T_1, T_2, T_3, …
- Therefore,
- $S_n = T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n$
- $S_n = (T_1 + T_2 + T_3 + \ldots + T_{n-1}) + T_n$
- $S_n = S_{n-1} + T_n$
- $T_n = S_n - S_{n-1}$
- nth term = (sum of first n terms) – (sum of first (n-1) terms)

Arithmetic Progression:

- A sequence in which each term is obtained by adding a constant number to its preceding term is called an Arithmetic Progression (A.P.)
- The constant number is called the ‘common difference’
- E.g. 2, 6, 10, 14, 18, … is an arithmetic progression in which the common difference is 4.

Arithmetic Progression:

- Generally first term of an AP is denoted by ‘a’ and the common difference is denoted by ‘d’,
- First term of AP: $T_1 = a$
- Second term of AP: $T_2 = a+d$
- Third term of AP : $T_3 = a + 2d$
- n^{th} term of AP: $T_n = a + (n-1)d$
- Hence the general form of an AP can be written as a, a+d, a+2d, a+3d, a+4d, …

Sum of n terms of AP

- Let the AP be a, a+d, a+2d, a+3d, a+4d, …
- Therefore, sum of n terms of AP
- $S_n = a + (a+d) + (a+2d) + (a+3d) + \ldots + a + (n-1)d$
- Let l be the last term
- $S_n = n \cdot (a + l) / 2$
- $S_n = n \cdot [2a + (n-1)d] / 2$

Ex.1

- Find the required terms of the following series:
- i) 37, 33, 29, 25… (50^{th} term)
- ii) 7, 8 ½, 10, 11 ½, … (81^{st} term)

Solution:

- i) In the A.P. 37, 33, 29, 25…
- Therefore
- a= 37
- d= - 4
- n = 50
- $T_n = a + (n-1)d$
- $T_{50} = \ldots$
- = - 159

Solution:

- ii) In the A.P. 7, 8 ½, 10, 11 ½, …
- Therefore
- a= 7
- d= 3/2
- n = 81
- $T_n = a + (n-1)d$
- $T_{81} = \ldots$
- = 127

Ex.2

- The 4^{th} term of an A.P. is 19 and its 12^{th} term is 51, find its 21^{st} term.

Solution:

- For an AP
- $T_n = a + (n-1)d$
- 4^{th} term is 19
- $T_4 = a + 3d \Rightarrow a + 3d = 19$
- 12^{th} term is 51
- $T_{12} = a + 11d \Rightarrow a + 11d = 51$
- Solve above two equations
- d = _____ a = _____

- $d = 4, \quad a = 7$

- Now 21st term:
- $T_{21} = a + 20d$
- =____
- =87

Ex.3

- The 6th term of an AP is 47 and its 10th term is 75, find its 30th term.

Ex. 4

- Find the sum upto the required number of terms of the followings:
- i) 100, 93, 86, 79, … (upto 20 terms)
- ii) 7, 19/2, 12, 29/2, 17, …(upto 30 terms)

Solution:

- i) For AP 100, 93, 86, 79, …
- $a = 100$
- $d = -7$
- $n = 20$
- $S_n = n . [2a + (n-1)d] / 2$
- $S_{20} =$ ______
- = 670

- ii) For AP 7, 19/2, 12, 29/2, 17, …
- $a = 7$
- $d = 5 / 2$
- $n = 30$
- $S_n = n \cdot [2a + (n-1)d] / 2$
- $S_{30} =$ ______
- $= 2595 / 2$
- $= 1297.5$

Ex.5

- The 4^{th} term of an AP is 22 and its 10^{th} term is 52, find the sum of its 40 terms.

Solution:

- For an AP
- $T_n = a + (n-1)d$
- For n = 4,
- $T_4 = a + 3d \Rightarrow a + 3d = 22$ – (i)
- For n= 10,
- $T_{10} = a + 9d \Rightarrow a + 9d = 52$ – (ii)
- Solving (i) and (ii)
- a = _____ , d = ______

- $d = 5$, $a = 7$
- Now,
- $S_n = n \cdot [2a + (n-1)d] / 2$
- For $n = 40$,
- $S_{40} =$ _____

- $= 4180$

Ex.6

- The sum of 6 terms of an AP is 57 and the sum of its 10 terms is 155, find the 20^{th} term.

Solution:

- Sum to n^{th} term:
- $S_n = n . [2a + (n-1)d] / 2$
- For n = 6,
- $S_6 = 57$
- $6 . [2a + 5 d] / 2 = 57$
- $2a + 5d = 19$ --- (i)
- For n = 10
- $S_{10} = 155$
- $10 . [2a + 9 d] / 2 = 155$
- $2a + 9d = 31$ --- (ii)

- Solving (i) and (ii)
- a = _____ , d = _____

- Now
- $T_n = a + (n-1)d$
- For n = 20
- $T_{20} =$_____

- = 59

Ex.7

- For the AP $d = 4$, $l = 40$, $n = 12$
- find a and S_n.

Ex.8

- For the AP $n = 20$, $l = 86$, $S_n = 1260$
- Find a

Ex.9

- Find the 40^{th} term and sum of first 40 terms of the sequence 1, 3, 5, 7, …

Ex.10

- Obtain the sum of first n natural numbers and hence find the sum of first 50 natural numbers.

Solution:

- First n natural numbers are
- 1, 2, 3, 4, 5, ….., n.
- Obviously this is an AP whose first term is 1 and common difference is also 1.
- $S_n = n \cdot [2a + (n-1)d] / 2 =$____
- $S_n = n (n+1) / 2$
- To obtain sum of first 50 natural numbers
- Put n = 50
- $S_{50} =$ ____
- =1275

Ex.11

- Find the sum of all natural numbers exactly divisible by 11 lying between 1 and 580.

Solution:

- The natural numbers divisible by 11, lying between 1 and 580 are
- 11, 22, 33, …, 572
- This is an AP with the first term a = 11,
- Common difference d = 11
- And last term $l = 572$
- $l = T_n = a + (n-1)d = 572$
- $11 + (n-1)\,11 = 572$

- n = _____
- n = 52

- Now $S_n = n \cdot (a + l) / 2$
- $S_{52} =$ _____

- $S_{52} = 15158$

Ex. 12

- Three numbers are in AP. Their sum and product are 39 and 2080 respectively. Find the numbers.

Solution:

- Let the numbers be a – d, a, a + d
- Their sum is 39
- $(a-d) + a + (a+d) = 39$
- $3a = 39$
- $a = 13$
- Their product is 2080
- $(a-d)\, a\, (a+d) = 2080$
- $a\,(a^2-d^2) = 2080$

- $13(169 - d^2) = 2080$
- $169 - d^2 = 160$
- $d^2 = 169 - 160$
- $d^2 = 9$
- $d = 3$ or $d = -3$
- Therefore required numbers are
- $a - d, a, a + d$
- If $d = 3$, $a = 13$ => 10, 13, 16 or
- If $d = -3$, $a = 13$ = > 16, 13, 10

Ex.13

- Five numbers whose sum is 50 are in AP. If the 5^{th} number is 3 times the 2^{nd} number, find the numbers.

Solution:

- Suppose that numbers are
- $a - 2d, a - d, a, a + d, a + 2d$
- Their sum is 50
- $(a-2d) + (a-d) + a + (a+d) + (a+2d) = 50$
- $5a = 50$
- $a = 10$
- Now (5^{th} number)= 3 x (2^{nd} number)
- $a + 2d = 3(a - d)$
- $a + 2d = 3a - 3d$

- $a + 2d = 3a – 3d$
- $2d + 3d = 3a – a$
- $5d = 2a$
- $5d = 2(10)$
- $d = 20 / 5$
- $d = 4$
- Hence the numbers are
- $a – 2d, a – d, a, a + d, a + 2d$
- $a = 10$ and $d= 4$
- 2, 6, 10, 14, 18

Ex.14

- The sum of first 7 terms of an AP is 168 and the 11^{th} term is 59.
- Find the sum of its first 30 terms
- Also find the 30^{th} term.

Solution:

- In the given AP: $S_7 = 168$ & $T_{11}=59$.
- $S_n = n \,.\, [2a + (n-1)d] / 2$
- $S_7 = 7 \,.\, [2a + 6d] / 2$
- $168 = 7 \,.\, [a + 3d]$
- $a + 3d = 24$ --- (i)
- $T_n = a + (n-1)d$
- $T_{11} = a + 10d \Rightarrow a + 10\,d = 59$ --- (ii)
- Solving eq. (i) and (ii)
- a = ______ , d = _______

- $a = 9$ and $d = 5$
- Now T_{30} = ___ and S_{30} =___
- $T_{30} = 154$ and $S_{30} = 2445$

Exercise

- Q.1 Find the required terms of the following:
- i) 10, 14, 18, 22,… (30^{th} term)
- ii) 59, 56, 53, 50, … (17^{th} term)
- iii) 101, 99, 97, 95, … (51^{st} term)

Q.2

- The 12th term of an AP is 20 and its 32nd term is 60.
- Find its 40th term.

Q.3

- The 20th term of an AP is 30 and its 30th term is 20.
- Find its 50th term.

Q.4

- Find the sum of the following series:
- i) 5, 9, 13, 17, … (up to 10 terms)
- ii) 32, 28, 24, 20, … (upto 13 terms)
- iii) 10, 12 ½, 15, 17 ½ , … (upto 25 terms)

Q.5

- The 3^{rd} term of an AP is 9 and its 9^{th} term is 21, find the sum of its 40 terms.

Q.6

- The sum of first 12 terms of an AP is 28 and the sum of its first 28 terms is 12, find the sum of its first 40 terms.

Q.7

- How many terms of the series 2, 5, 8, 11,… will make the sum 610?

Q.8

- The sum of three numbers in AP is 27 and their product is 585 find the numbers?

Q.9

- The sum of three numbers in AP is 15 and their product is 80 find the numbers?

Q.10

- The sum of five numbers in AP is 30 and the product of the first and last is 20, find the numbers?